REMARK. In (I) the sign changes with either x or y, that is, at the ends of either axis; while in (II) the sign changes only at the ends of the major axis. The factor $ex+a$ vanishes only on the left-hand directrix; hence not in the ellipse.

Also solved by G. B. M. Zerr, and J. Scheffer.

246. Proposed by A. M. HARDING, Adjunct Professor, University of Arkansas, Fayetteville, Ark.

A pentagon $ABCDE$, formed of equal uniform heavy rods connected by smooth joints at their ends, is supported symmetrically in a vertical plane with A uppermost, and AB and AE in contact with two smooth pegs in the same horizontal line. Prove that if the pentagon is regular, the pegs must divide AB and AE each in the ratio $1+\sin(\pi/10):3\sin(\pi/10)$. Jeans' *Theoretical Mechanics*, page 112, number 13.

Solution by PROF. F. L. GRIFFIN, Ph. D., Williams College.

Denote by P the point of contact of AB with one peg, and let $AP=2m$ and $PB=2n$. Also let the weight of each rod by $2W$, and the resistance exerted by each peg be R.

Consider first the rod AB, making an angle of 36° with the horizontal. At A the reaction of AE is horizontal (say X_a); for if there were a vertical component, then AB would exert upon AE an oppositely directed vertical component, contrary to the hypothesis of symmetry. At B the reaction of BC is unknown; denote its components by X_b and Y_b. The other forces applied to AB are its weight $2W$, and the reaction R inclined 54° to the horizontal. Equating to zero the sums of the vertical components, horizontal components, and moments about P we obtain (since the distance from P to the midpoint of AB is $2m-(m+n)=m-n$).

(1), (2) $\qquad Y_b+R\sin54^\circ-2W=0, \quad X_a+X_b+R\cos54^\circ=0,$

(3) $\qquad X_b2n\cos54^\circ+Y_b2n\sin54^\circ+2W(m-n)\sin54^\circ-X_a2m\cos54^\circ=0.$

Now R is easily found by considering the pentagon as a whole, to which are applied three external forces, R, R and the weight $10W$. The vanishing of the vertical component gives

(4) $\qquad 2R\sin54^\circ-10W=0, \quad R=5W\csc54^\circ.$

To obtain X_b consider the rod BC, to which is applied at B the reaction of AB whose components are respectively $(-X_b)$ and $(-Y_b)$. Equating to zero the resultant moment about C, we have, since $BC=2(m+n)$ and is inclined 72° to the horizontal:

(5) $\qquad (-Y_b).2(m+n)\sin18^\circ-(-X_b)2(m+n)\cos18^\circ-2W(m+n)\sin18^\circ=0.$

Using (4) to solve successively (1), (5), and (2) we obtain

$$Y_b=-3W,\quad X_b=(W+Y_b)\tan 18^\circ=-2W\tan 18^\circ,$$
$$X_a=2W\tan 18^\circ-5W\cot 54^\circ.$$

Putting these values into (3) and dividing by $2W$ we get

$$(6)\qquad -2\tan 18^\circ.n.\cos 54^\circ-3W.n\sin 54^\circ+(m-n)\sin 54^\circ$$
$$+(5\cot 54^\circ-2\tan 18^\circ).m\cos 54^\circ=0.$$

Multiplying (6) by $\sin 54^\circ$, collecting m and n, and later employing the identities $2\sin^2 x=1-\cos 2x$, $2\cos^2 x=1+\cos 2x$, and $2\sin x\cos x=\sin 2x$, we obtain:

$$m[\sin^2 54^\circ-2\sin 54^\circ\cos 54^\circ\tan 18^\circ+5\cos^2 54^\circ]$$
$$=n[4\sin^2 54^\circ+2\sin 54^\circ\cos 54^\circ\tan 18^\circ],$$
$$m[3+2\cos 108^\circ-\sin 108^\circ\tan 18^\circ]=n[2-2\cos 108^\circ+\sin 108^\circ\tan 18^\circ],$$
$$\text{or,}\quad m[3-3\sin 18^\circ]=n[2+3\sin 18^\circ],$$
$$\text{whence,}\quad m:n=2+3\sin(\pi/10):3-3\sin(\pi/10).$$

The proposed result follows immediately from this one, if we recall from geometry that the half-side of the regular inscribed decagon$=R(\sqrt{5}-1)/4$, so that $\sin 18^\circ$ satisfies the equation

$$(4\sin x+1)^2=5 \text{ or } 4\sin^2 x+2\sin x=1.$$

For from this we obtain $\sin x(3\sin x+2)=1-\sin^2 x$, or $2+3\sin x:3(1-\sin x)::1+\sin x:3\sin x$.

PROBLEMS FOR SOLUTION.

ALGEBRA.

343. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

A, on contracting to execute a piece of work for \$300 and finding after working alone one day that he had finished but 1% of the entire work, engaged B to assist him at the beginning of the second day, with the understanding, that B on each day was to do 6% as much work as had been completed previously, while A each day was to do an amount of